MATH 20D Spring 2023 Lecture 14.

More Variation of Parameters and Reduction of Order

Outline

Last Time

- Consider an inhomogeneous ODE
$$y''(t) + p(t)y'(t) + q(t)y(t) = g(t). \tag{1}$$
where $p(t)$, $q(t)$, and $g(t)$ are continuous function defined on an interval I.

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- Let $y_1(t)$ and $y_2(t)$ be **linearly independent** solutions to the homogeneous equation corresponding to (1) and write

$$\mathrm{Wr}[y_1, y_2](t) = y_1(t)y_2'(t) - y_2(t)y_1'(t)$$

for the Wronskian of the solutions y_1 and y_2.

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Theorem

- *There exists a constant* $W_0 \neq 0$ *such that* $\mathrm{Wr}[y_1, y_2](t) = W_0 \exp\left(-\int p(t)dt\right)$

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Theorem

- *There exists a constant* $W_0 \neq 0$ *such that* $\mathrm{Wr}[y_1, y_2](t) = W_0 \exp\left(-\int p(t)dt\right)$
- *If* $v_1(t) = \int \frac{-g(t)y_2(t)dt}{\mathrm{Wr}[y_1,y_2](t)}$ *and* $v_2(t) = \int \frac{g(t)y_1(t)dt}{\mathrm{Wr}[y_1,y_2](t)}$ *then*

$$y_p(t) = v_1(t)y_1(t) + v_2(t)y_2(t)$$

is a particular solution to (1).

Contents

Example

Using variation of parameters, give a solutions to the initial value problem

$$y'' + y = \tan(t), \qquad y(0) = 0, \quad y'(0) = 0.$$

over the interval $t \in (-\pi/2, \pi/2)$.

Contents

- Given two linearly independent solutions to a homogeneous equation

$$y''(t) + p(t)y'(t) + q(t)y(t) = 0 \tag{2}$$

variation of parameters allows us to find a particular solution to any inhomogeneous equation $y''(t) + p(t)y'(t) + q(t)y(t) = g(t)$.

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Question

Is there a general method for constructing a pair of linearly independent solutions to the homogeneous equation $y''(t) + p(t)y'(t) + q(t)y(t) = 0$*?*

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Question

Is there a general method for constructing a pair of linearly independent solutions to the homogeneous equation $y''(t) + p(t)y'(t) + q(t)y(t) = 0$?

Theorem

Suppose $y_1(t)$ *is a non-zero solution to (2) defined on an interval* I. *Then*

$$y_2(t) = y_1(t) \int \frac{e^{-\int p(t)dt}}{y_1(t)^2} dt$$

is a second linearly independent solution to (2).

Example

Given that $y_1(t) = t$ is a solution to the equation

$$y'' - \frac{1}{t}y' + \frac{1}{t^2}y = 0, \qquad t > 0$$

use the reduction of order to determine a second linearly independent solution.